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# A Markovian Queueing Framework for Wireless Sensor Networks with Service Interruptions

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## Abstract

Wireless Sensor Networks (WSNs) has made significant advancements in various areas and are increasingly being integrated into larger IoT ecosystems, enabling seamless connectivity and data exchange between sensors enhancing the scalability and functionality allowing for diverse applications. This integration improves the reliability, adaptability, and effectiveness of WSNs, allowing for a wide range of applications such as smart cities, industrial automation, environmental monitoring, and healthcare systems. Despite these developments, energy efficiency and fault detection still continue to be significant considerations for WSN reliability. Timely detection and diagnosis of faults are essential to maintain network performance, prevent data loss, and reduce downtime. Duty cycling has emerged as a key strategy for energy-efficient operation in WSNs reducing idle consumption of energy while retaining adequate network responsiveness to detect and respond to issues. Duty cycling enables adaptive resource management, as active and sleep durations can be dynamically modified in response to network demand, sensor failure rates. The proposed methodology provides a comprehensive study aimed at understanding the interplay between energy consumption and system failures in WSNs with the implementation of queueing theory, where sensor nodes alternate between active and sleep states during the breakdown state to reduce power consumption. Using generating function technique explicit transient state probabilities of various stages of power management modes are computed to evaluate system performance metrics, such as response times, busy and idle state probabilities and recovery rates. Numerical findings reveal that well designed duty cycles can greatly enhance energy consumption while preserving fault detection capabilities, underlining the need of adaptive power management in ensuring reliable WSN operations.

**Keywords:** Transient solution, Power consumption, Duty cycle, Fault detection, Sensor nodes.

## 1 | Introduction

A Wireless Sensor Network (WSN) comprises spatially distributed autonomous sensor nodes that communicate wirelessly to monitor physical or environmental changes. These nodes are typically equipped

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with sensors to measure various parameters such as temperature, humidity, light, sound, motion, or pollutants. WSNs serve diverse applications across numerous domains, including environmental monitoring, industrial automation, healthcare, home automation, agriculture, and smart cities. The primary concern in WSNs is energy consumption, which is largely influenced by the sensor node's communication capabilities [1]. Various low-power design strategies, such as duty cycling, sleep modes, and energy-efficient protocols, are employed to prolong the operational lifetime of WSNs. However, they also pose several challenges, including limited power and energy resources, constrained computational capabilities, communication constraints (e.g., bandwidth, latency), and reliability issues in harsh or dynamic environments [2]. To address these challenges, research and development efforts focus on various aspects, including energy-efficient communication protocols, adaptive data processing, and aggregation techniques to enable pervasive sensing and monitoring capabilities, contributing to advancements in fields ranging from environmental conservation and resource management to healthcare and smart infrastructure.

Duty cycling, a power management technique used in WSNs to conserve energy, can also play a role in fault detection, enabling timely detection, localization, and response to anomalies while minimizing energy consumption and prolonging network lifetime. This approach results in effective fault detection without compromising energy efficiency [3]. The two main causes of energy waste are collision and passive listening. Duty cycling places nodes in a recurring sleep/wake state and adjusts sensor nodes' sleep and wake times to optimize network longevity and meet application requirements such as minimal latency or high throughput. It can also be integrated into fault recovery mechanisms to minimize energy consumption while resolving faults. For example, instead of keeping all nodes continuously active during fault recovery operations, only a subset of nodes may be selectively awakened to perform necessary tasks, reducing energy overheads and prolonging network lifetime. Since the node uses twice as much energy in wake mode as in sleep mode, turning off the transceiver when in sleep mode helps preserve energy. The duty cycle of each node is adjusted based on packet analysis and failure rates. With this method, a node lengthens its active period during periods of high traffic, reducing the time packets in the queue must wait. Since the node uses twice as much energy in wake mode as in sleep mode, turning off the transceiver when in sleep mode helps preserve energy.

Breakdowns in WSN may occur due to various factors such as hardware failures, environmental conditions, or communication errors. In fact, these breakdown rates vary, and the equipment providing service has different failure rates during both busy and idle periods. Idle breakdowns can affect the node's ability to wake up from sleep mode, transmit data, or respond to commands. Active breakdowns can disrupt the node's ability to perform its designated tasks effectively, leading to data loss or communication failures. In the unfortunate event of a system failure, such as a malfunction at service stations, all data packets stored in the sensor node are systematically purged. The system's duty cycle, a pivotal parameter governing its operational mode, is dynamically adjusted based on the frequency of system failures and the average queue length of pending data packets awaiting transmission. By strategically calibrating the duty cycle based on these factors, the system optimizes resource allocation, balancing the need for uninterrupted data flow with energy conservation objectives.

In WSNs, the duty cycling mechanism conserves energy by periodically switching between active and sleep modes. However, breakdowns can still occur during both the active (busy) and sleep (idle) periods of the duty cycle [4]. During the idle period, sensor nodes are in a low-power sleep mode to conserve energy. Integrating fault detection mechanisms into the duty cycle algorithm to detect various types of faults, including hardware failures and communication errors, and to identify abnormal behavior indicative of faults. If anomalies or faults are detected, appropriate actions such as triggering alarms, logging events, or initiating fault recovery procedures are initiated, and duty cycle parameters are adjusted based on fault severity and breakdown rates, resulting in extended network lifetime and minimizing packet delay, energy efficiency, fault detection accuracy, and network throughput. In the earlier works, it was frequently assumed that failures happen when the system is busy. That can't always be the case in real-world circumstances, though. A few academics have made contributions in this direction by creating Markovian models that account for failure rates during both busy and idle periods and are primarily concerned with determining steady-state probabilities. On the other hand, time dependence is important for system performance measurements when the system's state changes over time, as seen from a practical standpoint. We note that the transient study has received little attention in the research literature. To address this problem, we present a transient-state analysis of an M/M/1 queuing system with a duty-cycling power design strategy, ensuring that sensor nodes effectively balance energy efficiency with fault tolerance in the WSN. This method is characterized by considerable energy consumption from collisions and idle listening.

We propose an adaptive method that modifies the node's duty cycle based on the queue length and failure probabilities, along with transient probabilities calculated using the generating function technique. We integrate a Markovian M/M/1 queueing model and a duty-cycle algorithm to analyze energy consumption in the system's transient state under varying breakdown rates and several performance metrics in a WSN. The transient solution of the model is obtained by building and solving the governing equations of the system state probabilities using the generating function technique and the Laplace transform. The rest of the paper is structured as follows: Section 2 discusses the relevant literature, and Section 3 presents the mathematical formulation, along with a proposed duty cycle scheme, and derives transient and failure probabilities in subsections. To assess system performance and reliability metrics, an adaptive duty cycle scheme is proposed that adjusts the node duty cycle based on transient probabilities and breakdown rates, as discussed in Section IV. The graphical illustrations presented in Section V highlight various performance measures. The last section concludes with a future scope section, followed by references.

# 2 | Earlier Literatures

The research background in this area can broadly be classified into two main streams. The first stream focuses on developing mechanisms to explain and model sleep–wake behavior in queueing systems. In contrast, the second stream emphasizes the design of new predictors and control strategies highly relevant to the present investigation. In this context, we present some earlier works that are very relevant to our proposed investigation. Duty cycle algorithms play a crucial role in managing resources efficiently in queueing systems. These mechanisms apply duty-cycle approaches that are fixed, differential, or adaptive. Duty cycle algorithms are paramount for efficient resource management in queueing systems, especially in energy-constrained environments like WSNs. These methods commonly use fixed, differential, or adaptive duty cycle approaches. Mathematical techniques are frequently used to assess the efficacy of various duty cycle algorithms across a range of operational conditions. In general, a duty cycle algorithm divides time into active and inactive (sleep) periods, and the durations of these phases are adjusted based on system load, task priority, or other performance-related variables. A single-server Poisson queue, when subject to catastrophes, was analyzed to derive the transient solution using the generating function technique. Various Performance indices, such as average end-to-end delay, packet loss, throughput, and intermittency duration, have been derived using G/G/1/N queuing networks [5]. A repairable M/M/2 queueing model [6] with threshold policy to analyze the various switching modes in WSN to obtain a steady-state solution using the matrix-geometry approach. In practical WSN deployments, data retransmissions are inevitable, and the effect of retransmission delays on end-to-end delivery performance was investigated in [7]. [8] provides a comprehensive overview of Markovian queueing concepts and analytical methodologies. An adaptive low-duty-cycle approach developed in [9] addressed transmission delay issues in WSNs, resulting in a 27% improvement in energy efficiency, network longevity, and latency. Adaptive duty cycle solutions often adjust the node's duty cycle based on queue length, prioritizing energy efficiency over strict Quality-of-Service (QoS) criteria such as latency and throughput. Maintaining a low duty cycle has been recognized as an effective strategy to reduce energy consumption while maintaining acceptable delay constraints in WSNs [10], [11].

Multi-server queueing models with service interruptions and varying breakdown rates were analyzed in [12], [13], where failure probabilities and transient-state probabilities were obtained using generating function techniques.

Despite extensive studies, the existing literature has largely overlooked fault detection systems that account for service failures during both idle and busy periods, as well as repair activities. The proposed study addresses this gap by presenting a queueing-theoretic fault detection framework that combines service failures and repair rates to achieve stated performance requirements, thereby advancing and complementing previous research.

# 3 | Mathematical Formulation

Consider a WSN with sensor nodes deployed in an environment. We consider a single infinitely capable server with three modes: idle, busy, and breakdown. The data transmission process at each sensor node can be modeled as an M/M/1 queue. Data packets arrive at the sensor node according to a Poisson distribution at rate λ. Service times, exponentially distributed with a mean of 1, are processed and transmitted by the node and depart from the system. Each sensor node follows a duty cycle algorithm, alternating between an active state, in which it senses, processes, and transmits data; a sleep state, in which it conserves energy by turning off non-essential components; and a failure state, entered during a breakdown. Additionally, sensor nodes can be affected by system failures that may occur during both idle (γ1) and busy periods (γ2). After serving all events (data packets) in the busy state, the server enters an idle state for an arbitrary period. Disasters can occur at any point in the system's life, except for failure, when repair is considered. Events that arise during the outage can't be accommodated until the repairs are complete. When the server is idle, data frames are added to the queue. Entering a busy state enables the sensing to receive data frames from the idle state and to transfer them (service). Also, service occurs only during the server's busy period, not during idle periods. The server enters a failure state during breakdowns and returns to an idle state once the repair η process is completed, after which it resumes service.

Further, it is assumed that whenever a breakdown occurs, the server is sent for repair and resumes its idle state after the repair time η. We assume that breakdown occurs in both the idle and busy periods, and that service or transfer of data frames occurs only in the busy state, whilst it has been sensed and wielded in the idle state. To estimate various transient probabilities associated with different server states, the Kolmogorov set of equations has been formulated using the transition flow rates of the birth-death process specifying the proposed Markov model. In addition, duty cycles derived from probability analysis are developed to allocate resources effectively, rather than relying on fixed maintenance schedules, and to prioritize replacement efforts for nodes with high failure probabilities or unusual transient behavior.

## 3.1 | Governing Equations of the Proposed System

The differential-difference equations of the proposed model have been framed as:

$$\frac{dP_{00}(t)}{dt} = -(\lambda + \eta)P_{00}(t) + \gamma_1 P_{10}(t), n = 0. \quad (1)$$

$$\frac{dP_{0n}(t)}{dt} = -(\lambda + \eta)P_{0n}(t) + \lambda P_{0n-1}(t) + \gamma_2 P_{1n}(t), n \geq 1. \quad (2)$$

$$\frac{dP_{10}(t)}{dt} = -(\lambda + \gamma_1)P_{10}(t) + \eta P_{00}(t) + \mu P_{11}(t), n = 0. \quad (3)$$

$$\frac{dP_{1n}(t)}{dt} = -(\lambda + \mu + \gamma_2)P_{1n}(t) + \lambda P_{1n-1}(t) + \mu P_{1n+1}(t) + \eta P_{0n}(t), n \geq 1. \quad (4)$$

### 3.1.1 | Transient probabilities

Let Pin(t) = P [M (t) = i, N (t) = n] denote the transient state probabilities of the system and [M (t), N (t), t 0], with state space (i, n), i = 0, 1, Q, n = 0, 1, 2, . . . be a continuous time Markov chain.

Considering the system of *Eq. (3)* and *Eq. (4)* to evaluate $P_{1n}(t)$, we define the generating function as:

$$P_i(z, t) = \sum_{n=1}^{\infty} P_{in}(t)z^n, i = 0,1. \quad (5)$$

Considering the system of *Eq. (3)* and *Eq. (4)* to evaluate P1n(t) and after some algebraic manipulations we obtain

$$\frac{\partial P_1(z,t)}{\partial t} = \left[\lambda(z-1) + \gamma_2 + \mu\left(1-\frac{1}{z}\right)\right] P_1(z,t) - \left[\mu\left(1-\frac{1}{z}\right) - \gamma_1\right] P_{10}(t) + \eta P_0(z,t). \quad (6)$$

On integration the above equation becomes

$$P_1(z,t) = e^{\left[\left(\lambda z+\frac{\mu}{z}\right)-(\lambda+\mu+\gamma_2)\right]t} \int_0^t P_{10}(u) \times e^{\left[\left(\lambda z+\frac{\mu}{z}\right)-(\lambda+\mu+\gamma_2)\right](t-u)} du + \eta \int_0^t P_0(z,u) e^{\left[\left(\lambda z+\frac{\mu}{z}\right)-(\lambda+\mu+\gamma_2)\right](t-u)} du. \quad (7)$$

Using the standard properties of the Bessel function, α = 2√λμ and β = λ/μ.

Then, using modified Bessel function of first kind In(.) and the Bessel function properties we get

$$\exp\left(\lambda z + \frac{\mu}{z}\right)t = \sum_{n=-\infty}^{\infty} (\beta z)^n I_n(\alpha t).$$

Now, substituting the above in *Eq. (7)* and comparing the coefficients of zn on either sides of equation by taking summation for all possible values of n = 1, 2, . . . we obtain

$$P_{1n}(t) = e^{-(\lambda+\mu+\gamma_2)} t I_n(\alpha t)\beta^n + (\mu-\gamma_1)\beta^n \times \int_0^t P_{10}(u) I_n\big(\alpha(t-u)\big) e^{(\lambda+\mu+\gamma_2)](t-u)} du - \mu\beta^{n+1} \int_0^t P_{10}(u) I_{n+1}\big(\alpha(t-u)\big) e^{(\lambda+\mu+\gamma_2)(t-u)} du. \quad (8)$$

Taking the Laplace transform of the above equation and simplifying it results in

$$P_{1n}(s) = \frac{1}{\sqrt{\ell^2-\alpha^2}}\left(\frac{\ell-\sqrt{\ell^2-\alpha^2}}{2\mu}\right)^n + \frac{\mu-\gamma_1}{\sqrt{\ell^2-\alpha^2}}\left(\frac{\ell-\sqrt{\ell^2-\alpha^2}}{2\mu}\right)^n P_{10}(s) - \frac{\mu}{\sqrt{\ell^2-\alpha^2}}\left(\frac{\ell-\sqrt{\ell^2-\alpha^2}}{2\mu}\right)^{n+1} P_{10}(s) + \frac{\eta}{\sqrt{\ell^2-\alpha^2}} \sum_{r=0}^{n-1} P_{0r}(s)\left(\frac{\ell-\sqrt{\ell^2-\alpha^2}}{2\mu}\right)^{n-r}, \quad (9)$$

where l = s + λ + μ + γ2

As Pi(z, t) does not contain negative powers of z, by replacing the right-hand side of *Eq. (8)* with n and the left-hand side with zero, we get

$$0 = e^{-(\lambda+\mu+\gamma_2)t} I_n(\alpha t)\beta^n + (\mu-\gamma_1)\beta^{-n} \int_0^t P_{10}(u) I_n\big(\alpha(t-u)\big) e^{-(\lambda+\mu+\gamma_2)(t-u)} du - \mu\beta^{-(n+1)} \int_0^t P_{10}(u) I_{n+1}\big(\alpha(t-u)\big) e^{-(\lambda+\mu+\gamma_2)(t-u)} du + \eta \int_0^t \sum_{r=0}^{\infty} P_{0r}(u)\beta^{-(n+r)} I_{n+r}\big(\alpha(t-u)\big) e^{-(\lambda+\mu+\gamma_2)(t-u)} du. \quad (10)$$

Using the property I−n(x) = In(x) and performing algebraic simplification, we obtain

$$\begin{aligned}P_{1n}(s) = {} & \frac{\beta^n}{\sqrt{l^2-\alpha^2}}\left(\frac{l-\sqrt{l^2-\alpha^2}}{2\mu}\right)^n + \frac{(\mu-\gamma_1)\beta^n}{\sqrt{l^2-\alpha^2}}\left(\frac{l-\sqrt{l^2-\alpha^2}}{2\mu}\right)^n P_{10}(s) \\ & - \frac{\mu\beta^{n+1}}{\sqrt{l^2-\alpha^2}}\left(\frac{l-\sqrt{l^2-\alpha^2}}{2\mu}\right)^{n+1} P_{10}(s) \\ & + \frac{\eta}{\sqrt{l^2-\alpha^2}}\sum_{r=0}^{n-1} P_{0r}(s)\beta^{n-r}\left(\frac{l-\sqrt{l^2-\alpha^2}}{2\mu}\right)^{n-r} \\ & + \frac{\phi_n(s)}{\sqrt{l^2-\alpha^2}}\left[1+(\mu-\gamma_1)P_{10}(s) - \sqrt{l^2-\alpha^2}P_{10}(s) - \frac{l-\sqrt{l^2-\alpha^2}}{2}P_{10}(s)\right].\end{aligned} \tag{11}$$

Taking the Laplace inverse of the previous equation provides an explicit expression for P1n(t)

$$\begin{aligned}P_{1n}(t) = {} & \beta^n[I_{n-1}(\alpha t) - I_{n+1}(\alpha t)]e^{-(\lambda+\mu+\gamma_2)t} + (\mu-\gamma_1)\beta^{-n}\left(P_{10}(t) * I_n(\alpha t)e^{-(\lambda+\mu+\gamma_2)t}\right) \\ & - \mu\beta^{-(n+1)}\left(P_{10}(t) * I_{n+1}(\alpha t)e^{-(\lambda+\mu+\gamma_2)t}\right) + \left(\phi_n(t) * I_0(\alpha t)e^{-(\lambda+\mu+\gamma_2)t}\right) \\ & - (\mu-\gamma_1)\left(P_{10}(t) * \phi_n(t) * I_0(\alpha t)e^{-(\lambda+\mu+\gamma_2)t}\right) + \left(\phi_n(t) * P_{10}(t)\right) \\ & + \frac{\alpha}{2}\left(\phi_n(t) * P_{10}(t) * I_1(\alpha t)e^{-(\lambda+\mu+\gamma_2)t}\right) \\ & + \eta\sum_{r=0}^{n-1} \beta^{n-r}P_{0r}(t)I_{n-r}(\alpha t)e^{-(\lambda+\mu+\gamma_2)t}.\end{aligned} \tag{12}$$

Similarly, comparing the co-efficient of z−1 on both sides to compute P10(t) and using the Bessel functions identity we get

$$\begin{aligned}0 = {} & e^{-(\lambda+\mu+\gamma_2}tI_1(\alpha t)\beta^{-1} + (\mu-\gamma_1)\beta^{-1} \times \int_0^t P_{10}(u)I_1\alpha(t-u)e^{-(\lambda+\mu+\gamma_2)(t-u)}du \\ & - \mu\beta^{-(n+1)}\int_0^t P_{10}(u)I_0\big(\alpha(t-u)\big)e^{-(\lambda+\mu+\gamma_2)}(t-u)du.\end{aligned} \tag{13}$$

Taking Laplace transform of the above equation we get

$$\begin{aligned}P_{10}(s) = {} & \frac{1}{2\lambda\mu}\sum_{k=0}^{\infty} (-1)^{k+1}\left(\frac{\mu-\gamma_1}{2\lambda\mu}\right)^k\left(l-\sqrt{(l)^2-\alpha^2}\right)^{k+1} \\ & + \frac{\eta}{2\lambda\mu}\sum_{k=0}^{\infty} (-1)^{k+1}\left(\frac{\mu-\gamma_1}{2\lambda\mu}\right)^k\left(l-\sqrt{(l)^2-\alpha^2}\right)^{k+1} \\ & \times\sum_{r=0}^{\infty} P_{0r}(s)\left(\frac{l-\sqrt{(l)^2-\alpha^2}}{\alpha\beta}\right)^r.\end{aligned} \tag{14}$$

Now, inverting the resultant derived above results in

$$\begin{aligned}P_{10}(t) = {} & \frac{1}{2\lambda\mu}\sum_{k=0}^{\infty} (-1)^{k+1}\left(\frac{\alpha(\mu-\gamma_1)}{2\lambda\mu}\right)^k \\ & \times [I_k(\alpha t) \\ & - I_{k+2}(\alpha t)]e^{(\lambda+\mu+\gamma_2)t+\frac{\eta\alpha}{2}\sum_{k=0}^{\infty}(-1)^{k+1}\left(\frac{\alpha(\mu-\gamma_1)}{2\lambda\mu}\right)^k\times[I_k(\alpha t)-I_{k+2}(\alpha t)]e^{(\lambda+\mu+\gamma_2)t}} \\ & * \sum_{r=0}^{\infty} \beta^{-r}P_{0r}(t) * [I_{r-1}(\alpha t) - I_{r+1}(\alpha t)]e^{(\lambda+\mu+\gamma_2)t}.\end{aligned} \tag{15}$$

## 3.2 | Evaluation of P00(t)

Idle State Probabilities $P_{00}(t)$. The idle-state probability, $P_{00}(t)$, represents the probability that the server is idle and no customers are present in the system at time t. To derive $P_{00}(t)$, we first apply the Laplace transform to *Eq. (1)*, yielding

$$(s+\lambda+\eta)P_{00}(s)=\gamma_1 P_{10}(s), \tag{16}$$

where P10(s) is the Laplace transform of the probability that the server is busy with zero customers.

Substituting the previously obtained expression for P10(s) from *Eqs. (14)-(16)* into we obtain

$$\begin{aligned} P_{00}(s) = {} & \frac{\gamma_1}{2\lambda\mu}\sum_{m=0}^{\infty}\sum_{j=0}^{m}(-1)^m\binom{m}{j}\left(\frac{\eta}{2\lambda\mu}\right)^{m-j}\zeta^j\left(\frac{1}{s+\lambda+\eta}\right)^{m+1}\phi_1^k(s) \\ & \times\left[\sum_{k=0}^{\infty}(-1)^{k+1}\left(\frac{\mu-\gamma_1}{2\lambda\mu}\right)^k\left(l-\sqrt{l^2-\alpha^2}\right)^{k+1}\right]^{m-j+1} \\ & \times\left[\sum_{r=0}^{\infty}\phi_r(s)\left(\frac{l-\sqrt{l^2-\alpha^2}}{\alpha\beta}\right)^r\right]^{m-j}. \end{aligned} \tag{17}$$

Taking the Laplace inverse of the above results in an explicit time-domain expression for the idle-state probability

$$\begin{aligned} P_{00}(t) = {} & \frac{\gamma_1}{2\lambda\mu}\sum_{m=0}^{\infty}\sum_{j=0}^{m}(-1)^m\binom{m}{j}\left(\frac{\eta}{2\lambda\mu}\right)^{m-j}\zeta^j e^{-(\lambda+\eta)t}\frac{t^m}{m!}\phi_1^k(t) \\ & *\left[\sum_{k=0}^{\infty}(-1)^{k+1}\left(\frac{\alpha(\mu-\gamma_1)}{2\lambda\mu}\right)^k\left(I_k(\alpha t)-I_{k+2}(\alpha t)e^{(\lambda+\eta)t}\right)\right]^{*(m-j+1)} \\ & *\left\{\sum_{r=0}^{\infty}\beta^{j-m}[I_{r-1}(\alpha t)-I_{r+1}(\alpha t)]e^{(\lambda+\eta)t}*\phi_1^r(t)\right\}^{*(m-j)}, \end{aligned} \tag{18}$$

where $*(m-j+1)$ and $*(m-j)$ represent $(m-j+1)$- fold and $(m-j)$- fold convolutions, respectively.

The expressions derived for P00(t) captures the transient behavior of the server when it is idle, incorporating both server breakdowns ($\gamma 1$) and repairs ($\eta$) together with the transient solutions P1n(t) from *Eq. (12)* and *Eq. (15)*, as these solutions are essential for evaluating various performance metrics expected number of idle periods and influence of sensor or server failures on authentication request processing.

# 4 | System Performances

A numerical study of the proposed model is performed using MATLAB, and the system performance is illustrated through graphical representations. We analyze and present significant performance measures and energy consumption by grouping the system parameters and assuming a range of values to show the effect of breakdown rates in the idle and busy states. System parameters over a specific range reveal the effects of breakdown rates in idle ($\gamma 1$) and busy ($\gamma 2$) states on key performance metrics. Assuming the parameter values as arrival rate $\lambda = 2.5$ packets/sec, service rate $\mu = 3.5$ packets/sec, breakdown rates $\gamma 1 = 0.01$, and $\gamma 2 = 0.15$, and repair rate $\eta = 0.2$.

The system size gradually increases with higher repair rates, as shown in *Fig. 1*, indicating faster recovery from failures and greater system availability. As $\eta$ increases, service continuity improves and idle periods decrease. The system size grows as more customers (data frames) join, tends to stabilize, and increases gradually as the repair rate increases. These findings show that the system is sensitive to repair and breakdown rates, emphasizing the importance of optimizing duty cycles and repair processes to balance energy efficiency and dependability. The busy-state probability graphs have been plotted for different values of n in Fig. 2, illustrating the probability distribution over time for arrivals λ, breakdowns γ1, and service rate μ. As shown in the figure, there is a steady increase in P14(t) with λ = 2.0, μ = 3.5, and γ2 = 0 for different values of n (4, 5, 6, 7). The observed rise and fall patterns over time suggest that the system remains stable, indicating an effective balance between arrival and service rates in the absence of breakdowns.

The shifts between the three states, idle, busy, and failure, in *Fig. 3*, illustrate the exemplary impact of system features on the suggested model. As there are no production jobs available, the probability exhibits a sharp initial fall in the idle state. Because arrivals and service completions are balanced, the probability in the busy state increases steadily, reflecting active system operation. In contrast, in the failure state, operational reliability declines sharply as frequent breakdowns raise the failure probability. From *Fig. 4*, it was found that the various system probabilities decrease during the breakdown occurring during the busy period.

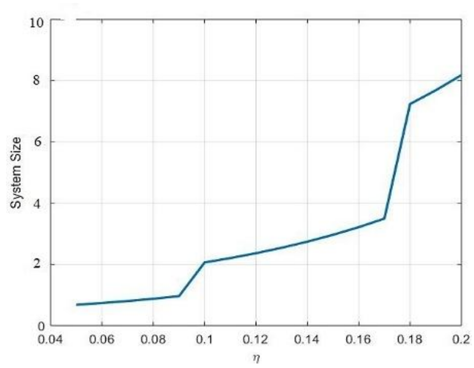


**Fig. 1. System size vs repair rate.**

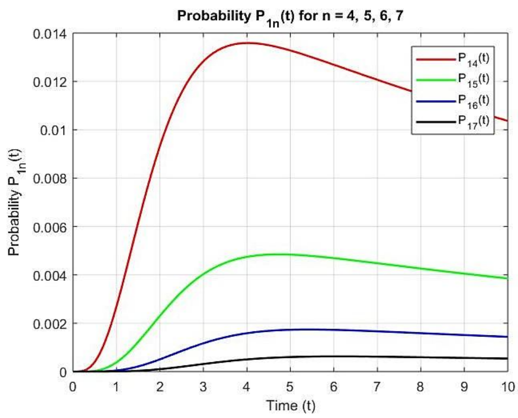


**Fig. 2. Busy state probabilities vs time.**

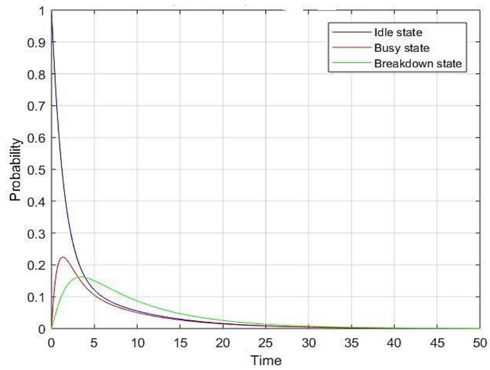


**Fig. 3. Server states Vs time.**

The key objective of this study is to examine energy consumption in WSNs and to assess duty-cycle-based power management solutions. *Fig. 5* and *Fig. 6* illustrate how adaptive duty cycling minimizes energy consumption while maintaining system responsiveness. *Fig. 5* compares breakdowns in idle and busy modes, confirming expected system behavior and verifying the proposed technique. The findings show that adjusting the active and sleep times improves energy efficiency, increases network lifetime, and provides effective fault detection.

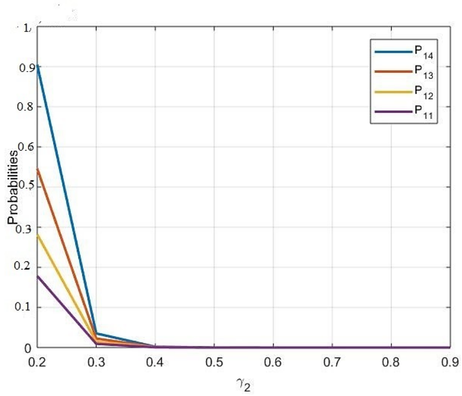


**Fig. 4. Probabilities Vs breakdown.**

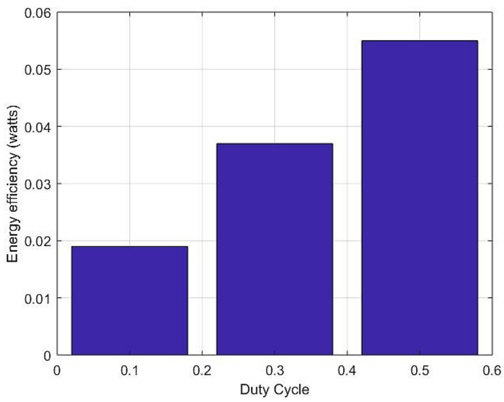


**Fig. 5. Energy consumption vs duty cycle.**

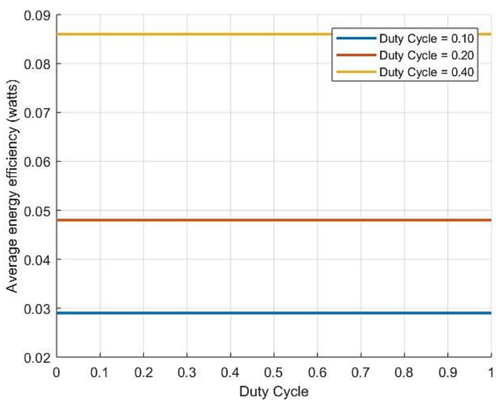


**Fig. 6. Average energy consumption Vs DC.**

*Fig. 7* shows the WSN's cumulative energy consumption as a function of time and duty cycle. Lower duty cycles reduce energy but may limit responsiveness, whereas higher duty cycles consume more energy due to longer active times. The highlighted range of 0.5-0.7 represents the optimal duty cycle, yielding significant energy savings while still processing authentication requests on time. Higher repair rates ($\eta$) increase system availability, while higher breakdown rates ($\gamma 1$, $\gamma 2$) increase idle periods. This highlights the necessity to optimize duty cycles and repair mechanisms for efficient and dependable operation.

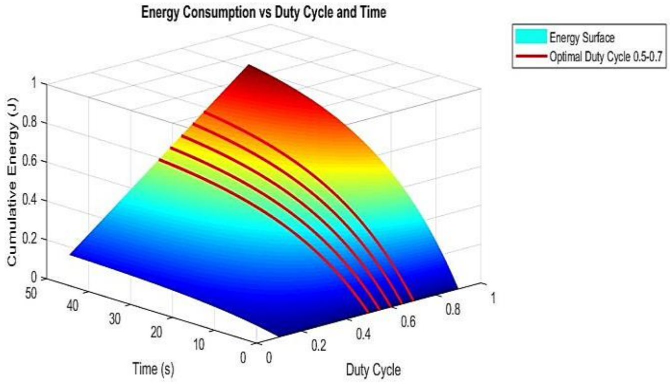

Fig. 7. Cumulative energy Vs DC and time.

# 6 | Duty Cycle Algorithm

The proposed duty cycle algorithm dynamically adjusts the active and sleep periods of sensor nodes based on breakdown rates in idle and busy periods.

λ: arrival rate of data packets.

μ: service rate of data packets.

γ1: breakdown rate during the idle period.

γ2: breakdown rate during the busy period. η: Repair rate of the system.

Dactive: active duty cycle duration. Dsleep: Sleep duty cycle duration.

Algorithm steps:

I. Initialize parameters

– *Set initial duty cycle dactive and dsleep.*
– *Initialize breakdown rates γ1 (idle) and γ2 (busy).*
– *Initialize state probabilities P00(0), P0n(0), P10(0), and P1n(0).*

II. Start algorithm

– *Begin the main loop.*

III. Main loop

– *Check system state, observe the system state (M (t), N (t)), where M (t) denotes the server state and N (t) denotes the number of authentication requests in the system.*

IV. Active period

– *When the system is in the active period:*
– *Process incoming authentication requests according to service rate μ.*
– *Continuously monitor the system for breakdown events.*

If failure is detected,

– *Immediately suspend service operations.*

*– Update the state probabilities using the governing differential equations.*
*– Transition the server to the idle state and enter the sleep period.*

V. Check for failures:

Idle failure,

*– If the server is idle (M(t) = 0), evaluate the breakdown events at rate $\gamma 1$.*
*– Update idle-state probabilities P00(t) and P0n(t).*

Busy failure,

*– If the server is busy (M(t) = 1), evaluate the breakdown events at rate $\gamma 2$. Suspend ongoing service.*
*– Update busy-state probabilities P10(t) and P1n(t).*

VI. Repair process

*– Initiate the repair process with rate $\eta$. Upon completion of repair, if the queue is non-empty, transition the server to the busy state; otherwise, retain the idle state.*

VII. Adjust duty cycle

- *Compute the busy-state probability using the system equations. Adjust duty cycle durations based on the probabilities of being in a busy state.*
- *Modify Dactive and Dsleep based on the busy-state utilization and system state. Increase Dactive when the busy-state occupation probability increases.*
- *Increase Dsleep when the idle-state occupation probability dominates.*

VIII. Sleep period

*– System enters sleep mode.*
*– Switch sensor nodes to sleep mode to reduce energy consumption.*
*– Monitor arrivals, breakdown events, and repair completion.*
*– Monitor the system state and prepare for the next active period. Continue monitoring for system faults.*

IX. Repeat

*– Return to Step 3.*

X.

End algorithm

## 7 | Conclusion

The mathematical formulation is carried over to derive the transient state probabilities using the Laplace transform technique. We compute the probability distributions of the system in idle and busy periods. The M/M/1 queue model can provide insights into the system's efficiency and performance. On the other hand, the duty cycle algorithm can shed light on the energy consumption patterns at the node level. Considering factors such as queuing delays and breakdowns, which in turn help in adjusting the duty cycle. The energy consumption of the sensor nodes can be optimized based on these factors, such as data traffic patterns, communication requirements, and power constraints. By combining both approaches, a thorough comprehension of WSN energy consumption has been achieved. This integrated approach can help design and optimize WSNs for energy-efficient operation while meeting performance requirements. The proposed methodology focuses on ensuring energy savings and minimizing delay by optimizing the active time and breakdowns. This paper's mathematical analysis and numerical examples demonstrate the validity of the proposed system, emphasizing its critical role in energy optimization in Wsn and its contribution to the network's optimal performance. Current duty cycle algorithms often rely on fixed parameters. Future work can explore adaptive duty cycling approaches that continuously adjust duty cycles in response to changes in environmental conditions.

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# Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

# Conflicts of Interest

The authors declare no conflicts of interest.

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